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University of Wisconsin, Department of Sociology
Sociology 376: Mathematical Models of Social Systems
Spring 2013

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8128B Social Science
Office hours: Friday 9:30-11:30 AM or by appointment

Course Objectives. This course provides an introduction to mathematical models of *social process*, focusing especially on Markov chains and dynamical systems models. Students will learn how to analyze these types of models in order to determine their short-run dynamics and long-run equilibria. Students will make extensive use of mathematical software (Matlab) to compute numerical examples and perform simple simulation analyses. Examples will address a wide range of sociological topics including social mobility, demography, network formation, social influence, cultural evolution, social movements, and residential segregation. This course complements Soc 375 (Introduction to Mathematical Sociology) which explores mathematical models of *social structure*, focusing on social network analysis and related methods.

Prerequisites. Formally, the prerequisite is Soc 375 or Math 340 or permission of the instructor. But in terms of mathematical preparation, the only essential prerequisite is some familiarity with matrix algebra. [Soc 375 provides a brief review of matrix algebra that is sufficient for the present course. Students who already know matrix algebra do not need to know social network analysis (or any other topics covered in Soc 375) to enroll in the present course.] Some knowledge of calculus would be helpful, but the course is intended to be accessible to students without calculus. [While more advanced courses on dynamical systems would require calculus, we will make use of graphical or computational methods whenever possible.] Students who have taken courses covering Markov chains (e.g., Math 331 or 632) and dynamical systems (e.g., Math 415) will already know the relevant mathematics, but will (hopefully) find interesting the social science applications. Students with some background in computer programming may have an advantage. For students who already know matrix algebra, there is no sociology prerequisite, so the course is well-suited for math or science majors.

Evaluation. Grades will be based on two exams (each worth 1/3 of the grade) and weekly problem sets (worth the final 1/3). The midterm exam will be held during class on **Thursday, March 14**; the final exam will be held during exam week on **Friday, May 17, 5:05 PM -7:05 PM**.

Exams. Past exams (with solutions) are posted on my website: www.ssc.wisc.edu/~jmontgom . These old exams are an importance resource for learning the course material, and students are strongly encouraged to work through these problems as we go along. (Don't wait until the night before the exam!) Students should always bring calculators to the exams. Graphing calculators (which can multiply matrices) are permitted but not necessary.

Problem sets. Problem sets will be assigned approximately once per week. Problem-set questions are usually more complicated than test questions, and often require the use of Matlab software (see below). Problem sets will be graded on a three-point scale, corresponding roughly to full credit (3), a good-faith effort (2), a bad-faith effort (1), and no effort (0).

Software. In this course, we make extensive use of the mathematical software package Matlab. While students who have taken Soc 375 will already know Matlab, the present course is intended to be self-contained for students who are not familiar with this software. However, students who do yet know Matlab **must be willing to learn**. Knowledge of Matlab will be necessary to follow the lectures (which are often supplemented with Matlab handouts) and to complete the problem sets. We will spend two class periods in the Social Science Micro Lab learning this software, **but some students may need to spend additional time on their own with the Matlab tutorial to become proficient**. Students can use this software in the Social Science Micro Lab or else access it online through the Social Science Computing Center website.

Readings. Four books have been ordered for this course:

Elizabeth S. Allman and John A. Rhodes (2004) *Mathematical Models in Biology: An Introduction.* Cambridge.
Ian Bradley and Ronald L. Meek (1986) *Matrices and Society*. Pelican.
Thomas C. Schelling (1978) *Micromotives and Macrobehavior*. Norton.
David McMahon (2007) *MATLAB Demystified*. McGraw-Hill.

Although the Bradley and Meek book is out of print, photocopies will be available at the University Book Store. The remaining readings will be posted as pdf files at the course site on Learn@UW (accessed through the UW homepage, or directly at https://learnuw.wisc.edu). Please note that readings from the four books listed above (indicated by a bullet point • on the reading list below) are not posted on Learn@UW. You need to buy those books.

Lecture notes. I am slowly turning my lecture notes into draft textbook chapters. These draft chapters are posted on my website at http://www.ssc.wisc.edu/~jmontgom/376textbook.htm . Please note that these draft chapters are preliminary and incomplete. They are not a perfect substitute for the doing the readings and attending the lectures.

Honors credit. Undergraduates automatically receive honors credit for this course.

Additional resources. Students interested in going beyond the course materials might see the website www.socdynamics.org for other syllabi and software.

Changes in the schedule. The tentative course outline is given below. As already indicated, the exam dates are fixed. The precise content of each exam will be announced in class before the exam. More generally, announcements of changes in course material and procedures may from time to time be made in class and students will be responsible for the changes whether present or not.

Course outline.

0. Introduction to Matlab

Some instruction will be given in class. However, students not already familiar with Matlab are encouraged to read

● McMahon, Chs 1-4, "The MATLAB Environment" and "Vectors and Matrices" and "Plotting and Graphics" and "Statistics and an Introduction to Programming in MATLAB."

Another helpful (longer but more thorough) introduction to Matlab is provided by the built-in tutorial. From the main Matlab window, click Help → Product Help to open the Help Navigator. In the Contents window, Expand the section on MATLAB, and then Getting Started. I encourage you to read all of the "Introduction" and "Matrices and Arrays" sections, the beginning of the "Graphics" section (covering 2D graphics), and then the "Programming" section.

1. Linear Systems

Markov chains

•Bradley and Meek, Ch 6 (“The Simple Mathematics of Markov Chains”) and Ch 7 (“Models of Mobility”)

John G Kemeny, J L Snell, and G L Thompson (1966), Ch 4.13 (“Markov Chains”), pp 194-201, and Ch 5.7 (“Application of Matrix Theory to Markov Chains"), pp 271-281, in *Introduction to Finite Mathematics*, Prentice-Hall.

Differential reproduction

•Allman and Rhodes, Ch 2.1 (“Linear Models and Matrix Algebra”) and Chs 2.3–2.4 (“Eigenvectors and Eigenvalues”)

Samuel Preston and Cameron Campbell (1993) “Differential Fertility and the Distribution of Traits: The Case of IQ,” *American Journal of Sociology* 98(5): 997-1043 [includes comment by Coleman].

Absorbing chains

•Bradley and Meek, Ch 8 (“The Mathematics of Absorbing Markov Chains”) and Ch 9 (“Everywhere Man is in Chains”)

John G Kemeny, J L Snell, and G L Thompson (1966), Ch 5.8 (“Absorbing Markov Chains”), pp 282-291, in *Introduction to Finite Mathematics*, Prentice-Hall.

Network formation

Thomas Fararo and John Skvoretz (1986) “E-State Structuralism: A Theoretical Method,” *American Sociological Review* 51:591-602.

Evolution of conventions

H Peyton Young (1996) “The Economics of Convention,” *Journal of Economic Perspectives* 10:105-22.

Herbert Gintis (2000), Ch 10 (“Markov Economies and Stochastic Dynamical Systems,”), pp 220-236, in *Game Theory Evolving*, Princeton.

Strategic network formation

Matthew Jackson and A Watts (2002) “The Evolution of Social and Economic Networks,” *Journal of Economic Theory* 106:265-95.

Influence networks

John G Kemeny, J L Snell, and G L Thompson (1966), Ch 7.2 (“Equivalence Classes in Communication Networks”), pp 394-406, in *Introduction to Finite Mathematics*, Prentice-Hall.

Noah Friedkin and Eugene Johnsen (1997) “Social Positions in Influence Networks,” *Social Networks* 19:209-222

Demography

•Bradley and Meek, Ch 10 (“The Seven Ages of Man and Population Problems”)

•Allman and Rhodes, Ch 2.2 (“Projection Matrices for Structured Models”)

2. Non-linear systems

Logistic growth model

•Allman and Rhodes, Ch 1 ("Dynamic Modeling with Difference Equations")

Threshold models

•Schelling, Ch 3 ("Thermostats, Lemons, and Other Families of Models")

Mark Granovetter (1978) "Threshold Models of Collective Behavior," *American Journal of Sociology* 83:1420-1443.

Neighborhood effects

see Chapter 13 of draft textbook at http://www.ssc.wisc.edu/~jmontgom/376textbook.htm

Dynamics of balance theory

T Antal, et al (2006) "Social Balance on Networks: The Dynamics of Friendship and Enmity," *Physica D* 224: 130-136.

Group interaction

Thomas Fararo (1989) Chs 2.9-2.10 (Simon-Homans model), pp 120-139, in *The Meaning of General Theoretical Sociology*, Cambridge.

Residential segregation

•Schelling, Ch 4 ("Sorting and Mixing: Race and Sex"), esp pp 155-166.

Mark Granovetter and Roland Soong (1988) "Threshold Models of Diversity: Chinese Restaurants, Residential Segregation, and the Spiral of Silence," *Sociological Methodology* 18:69-104.

Predator-prey models

•Allman and Rhodes, Ch 3 ("Nonlinear Models of Interactions")

Population dynamics of racial composition

James Montgomery (2011) "The Population Dynamics of Black-White-Mulatto Racial Systems," *American Journal of Sociology* 117:46-89.

Evolutionary game theory

Larry Samuelson (2002) "Evolution and Game Theory," *Journal of Economic Perspectives* 16:47-66.

Cultural evolution

Alberto Bisin and Thierry Verdier (2001) "The Economics of Cultural Transmission and the Dynamics of Preferences," *Journal of Economic Theory* 97:298-319.

James Montgomery (2010) "Intergenerational Cultural Transmission as an Evolutionary Game," *American Economic Journal: Microeconomics*, 2: 115-136.